Prediction of Gust Loadings and Alleviation at Transonic Speeds

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The transonic indicial theory is used to predict the effect of a gust on an airfoil at transonic speeds. The effect of operating two control surfaces is also modeled by the indicial method. The transonic indicial method is linear in a strained coordinate system, and superposition can be used. This allows the effects of an arbitrary gust and control surface deflection to be modeled simply if the indicial responses for the gust and each control surface are known. The computation time is small; therefore, an optimization technique can be used to determine the best control surface deflections to alleviate the gust loading.

Introduction

A PRESSING problem in designing a modern aircraft that flies at transonic speeds is prediction of gust response. Knowledge of the loading induced by a gust is necessary for the aerodynamicist to predict effects of the loading on wing performance. The structural engineer should also have this knowledge to design a sufficiently strong structure and, perhaps, to implement active controls to reduce the load. At present, the capability to predict gust loading accurately at transonic speeds does not exist. It is this problem that is addressed herein.

Aeroelastic calculations are frequently performed using linear theory for both the structure and aerodynamic loads. This leads to a relatively simple means of estimating flutter speeds, since linearity permits the use of the principle of superposition. Linear aerodynamic theory is really only applicable to the subsonic or supersonic speed regimes; the transonic regime is essentially nonlinear. This nonlinearity is associated with shock-wave formation, which is often accompanied in unsteady flow by complex shock oscillations. It is essential, therefore, to include the nonlinearities in such a way as to predict transonic flow phenomena. For the most general case, this requires the development of a three-dimensional transonic flow computer code for predicting unsteady effects, including viscous interactions between any shock waves that form and the boundary layer on the surface of the structure. Progress is being made toward the development of such a code,[1] but it is only half the problem, since the aerodynamic loads must interact with a dynamic structural model in order to predict aeroelastic phenomena. Accomplishing this interaction with a large and complex unsteady flow transonic code is not a trivial matter. Major advances in computer size and speed will be necessary before calculations of this sort can be routinely used in either the analysis or the design of aircraft.

A further problem is that conventional gust analysis may impose an upwash of finite length into the freestream. If this upwash is completely arbitrary, the flow is unlikely to be irrotational throughout the gust. Since all the available transonic solution methods are based on potential (irrotational) theory,[1,2] this presents difficulties in easily developing a gust load prediction method.

If the gust loading can be predicted, it is possible that some means to reduce the loading can be developed, perhaps by the deflection of a control surface. Also, due to the coupling of the mean steady and unsteady loading in a nonlinear transonic theory, certain wings may be more sensitive than others to gusts. The work reported here is concerned with a preliminary study of transonic gust loading and its alleviation. To retain simplicity at this stage, only two-dimensional problems are considered.

In recent years, the numerical simulation of steady transonic flow has reached a fairly advanced stage. For example, the two-dimensional codes based on the original algorithm of Murman and Cole[3] are frequently used in the aircraft industry. Three-dimensional variations of these codes for finite wings and wing/body combinations are also available, such as the codes of Bailey and Ballhaus[4] (transonic small disturbance) and Jameson and Caughey[5] (full potential). While these codes have certain deficiencies, they have been extremely useful in aircraft design.

The state-of-the-art regarding codes for unsteady flow is not as advanced as that for steady transonic flow. The direct integration method of Ballhaus and Goorjian[2] for low-frequency two-dimensional flows is a reliable tool. There have been several extensions of this theory to high-frequency flows, notably by Rizzetta and Chin[6] and the XTRAN2L code developed by Whitlow[7] at NASA Langley Research Center. Also, several preliminary attempts have been made to extend these two-dimensional algorithms to three-dimensional flow. Perhaps the most mature of these is the method developed by Rizzetta and Borland.[1] However, this three-dimensional code is very expensive to run on a computer at present. All these methods are based on potential theory.

In view of the computational expense in obtaining a transonic flow calculation, it is desirable to increase the usefulness of one calculation. To this end, the transonic indicial theory[8,9] can be used. The indicial theory depends on the validity of the principle of superposition. Such a method is not valid at transonic speeds because of the necessary nonlinearity of the governing equations that represent the flow with moving shock waves. In certain circumstances, however, unsteady transonic flow quantities can be represented by a linearized equation while the shock motion is still taken into account. Nixon[9] has shown that the transonic indicial method of Ballhaus and Goorjian[8] properly accounts for shock motion effects, when only integrated properties such as airfoil lift and moment coefficients are considered. The shock motion need not be explicitly included since it already appears in the calculation of the indicial response. A similar result applies for control-surface hinge moments if the shock does not oscillate across the hinge. When the shock oscillates across the hinge, the strained-coordinate method of Nixon[9] can be used to treat the shock motion. An application of the indicial approach is given in Ref. 10.

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In the indicial theory of Nixon,[9] the nonlinear problem is decoupled into two linear problems in which superposition is valid. Since superposition can be used, it is also possible to superpose the effects of various elements in an unsteady flow, such as flap movements or changes in angle of attack, provided the indicial response of each component in isolation is known. The capability to superpose is one of the most powerful features of the indicial method.

In connection with reducing loading at transonic speeds, it should be noted that a preliminary attempt was made by Ballhaus et al.[11] to study the effects of moving leading- and trailing-edge flaps on the loads on an airfoil.

The present work combines the methods of Kerlick and Nixon[10] and the XTRAN2L code to give the gust response of an airfoil. In addition, the effect of two control surfaces on the flow is included in the model. These methods are augmented by the use of the optimization code CONMIN in determining the control surface deflections to minimize the additional loading due to the gust. Because the computer time required by the indicial theory is greatly reduced, the optimization code is economical to use.

Basic Equations

The model equation used in the present work is the unsteady transonic small disturbance equation

$$\phi_{xx}+\phi_{yy}-2(M_\infty c^2/U_\infty)\phi_{xt}-M_\infty^2(c^2/U_\infty^2)\phi_{tt}=k\phi_x\phi_{xx} \qquad (1)$$

where $\phi(x,y,t)$ is the perturbation velocity potential; x and y Cartesian coordinates scaled by the airfoil chord c; and U_∞ the freestream velocity. The constant k is given by

$$k=[3+(\gamma-2)M_\infty^2]M_\infty^2 \qquad (2)$$

where γ is the ratio of specific heats. The boundary conditions used are those given by Ballhaus and Goorjian.[2] The tangency boundary condition can be written as

$$\phi_y(x,\pm 0,t)=\frac{\partial\bar{y}_s(x,\pm 0)}{\partial x}+\frac{\partial\hat{y}_s(x,\pm 0,t)}{\partial x}+\frac{c}{U_\infty}\frac{\partial\hat{y}_s(x,\pm 0,t)}{\partial t} \qquad (3)$$

where $\bar{y}_s$ and $\hat{y}_s$ represent the steady and unsteady components, respectively. For a control surface deflected at $\beta(t)$, $\hat{y}_s(x,\pm 0,t)$ is given by

$$\hat{y}_s(x,\pm 0,t)=-\beta(t)(x-x_h)H(x-x_h) \qquad (4)$$

where x_h is the location of the control surface hinge and $H(\ )$ is the step function.

In the present analysis, a semi-infinite gust is modeled by the boundary condition

$$\phi_y(x,\pm 0,t)=\frac{\partial\bar{y}_s(x,\pm 0)}{\partial x}+V_G(x,t) \qquad (5)$$

Thus,

$$\begin{aligned} v_G(x,t)&=|v_G|, & x&<U_\infty(t-t_0)\\ &=0, & x&\geq U_\infty(t-t_0) \end{aligned} \qquad (6)$$

where $|v_G|$ is the magnitude of the gust velocity; the gust starts at $t=t_0$ and travels with the freestream velocity U_∞. The gust model is similar to that of McCroskey and Goorjian.[12]

Piston Theory Limit for a Step Change in Normal Velocity

In the early phase of subsonic and supersonic flutter analysis, the piston theory limit was much used. Briefly, the supposition was that if the airfoil oscillated at very high frequencies, the velocity of the airfoil surface was an order of magnitude greater than the streamwise velocity. As a consequence, the linearized version of Eq. (1) becomes

$$\phi_{yy}=M_\infty^2(c^2/U_\infty^2)\phi_{tt} \qquad (7)$$

The solution is

$$\phi=f[y\pm(U_\infty c/M_\infty)t] \qquad (8)$$

For $y>0$, the positive sign is taken; for $y<0$, the negative sign is used. This is to ensure outgoing waves. The pressure coefficient in the approximation of Eq. (7) is given by

$$C_p(x,\pm y,t)=-2(c/U_\infty)\phi_t(x,\pm y,t) \qquad (9)$$

From Eq. (8),

$$\phi_y(x,y,t)=f'[y\mp(U_\infty c/M_\infty)t] \qquad (10)$$

For a step change in the normal velocity component of $-\alpha$, Eq. (10) gives

$$f'(0)=-\alpha \qquad (11)$$

Combining Eqs. (9) and (11) gives the result

$$C_p(x,\pm 0,0)=\mp(2\alpha/M_\infty) \qquad (12)$$

This is the initial pressure coefficient experienced by the airfoil due to a step change in angle of attack.

In the transonic equation [Eq. (1)], there is no mechanism to invalidate the basic premise of piston theory, namely, that streamwise velocities are small relative to the normal velocity. Hence, Eq. (12) is valid for transonic flow. Furthermore, since streamwise velocities contain the shock mechanism for Eq. (1), the initial movement of shock does not enter Eq. (7). Therefore, it is assumed that the shock movement does not start with the discontinuity that characterizes the pressure coefficient.

Transonic Indicial Method

In linear models of subsonic or supersonic flow, an alternative to expanding the governing equation in a Fourier series is to use an indicial function. An indicial function analysis depends on the governing equation being linear, which is not the case for transonic flow. However, if the amplitude of motion is small and shock waves are not generated or destroyed during the motion, a linear equation similar to that used in subsonic flow can be derived to model the unsteady motion. This is the basis for the work of Kerlick and Nixon.[10] The interested reader should refer to Ref. 10 for details of the method since, for brevity, only the final equations are given here.

The governing equations given by Kerlick and Nixon[10] are linear; thus, the principle of superposition can be used. In the present work, the unsteady flows due to a gust and two control devices are superposed. In this case,

$$\Delta x_s(t)=\sum_{i=1}^{3}\delta x_{s_{\epsilon_i}}(t)\epsilon_i(0)\int_0^t\delta x_{s_{\epsilon_i}}(\tau)\frac{d\epsilon_i(t-\tau)}{dt}d\tau \qquad (13)$$

$$C_{p_1}(x',y,t)=\sum_{i=1}^{3}C_{p_{\epsilon_i}}(x',y,t)\epsilon_i(0)+\int_0^t C_{p_{\epsilon_i}}(x',y,\tau)\frac{d\epsilon_i(t-\tau)}{dt}d\tau \qquad (14)$$

where the subscript i denotes a flow perturbation due to the ith device (either controls or the gust).

In Eqs. (13) and (14), $\delta x_{s_{\epsilon i}}(t)$ and $C_{p_{\epsilon i}}(t)$ are the indicial responses of the shock motion and the pressure coefficient, respectively, for the ith device. $\delta x_s(t)$ is the shock motion, $C_{p_1}(x',y,t)$ the pressure perturbation due to the motion, and ϵ_i the motion of the ith device. The actual pressure coefficient $C_p(x,y,t)$ is given by

$$C_p(x,y,t) = C_{p_0}(x',y,t) \times [1-\delta x_s(t) x_{1x'}(x')] + C_{p_1}(x',y,t) \tag{15}$$

where $x_1(x')$ is a known straining function and

$$x = x' + \delta x_s(t) x_1(x') \tag{16}$$

Curve Fits of Indicial Data

It is helpful in the numerical evaluations of Eqs. (13) and (14) if the indicial responses can be represented by analytic functions since the convolution integrals can then be evaluated analytically. In the present work, the indicial responses for both the pressure coefficient and the shock movement are approximated as

$$f(x,t) = a + b[\exp(-\alpha t)] + c[\exp(-\beta t)] \tag{17}$$

where $f(x,t)$ is a general indicial response, and a,b,c,α, and β are functions of x, in general. The reason for including a second exponential term in Eq. (17) is to allow for an inflection point on the indicial response curve. At $t=0$, $f(x,0)$ is represented by its piston theory limit $f_p(x)$. Thus,

$$f_p(x) = a + b + c \tag{18}$$

As $t \to \infty$, $f(x,t)$ must approach its steady-state value $f(x,\infty)$ corresponding to the step change in ϵ. Thus,

$$f(x,\infty) = a \tag{19}$$

Combining Eqs. (17-19) gives

$$f(x,t) = f(x,\infty)[1-\exp(-\alpha t)] + f_p(x)\exp(-\alpha t) + c[\exp(-\beta t) - \exp(-\alpha t)] \tag{20}$$

Apart from the initial jump predicted by piston theory, the indicial response is a smooth curve. Consequently, $[f(x,t) - f_p(x)]/f(x,\infty)$ can be approximated by

$$[f(x,t) - f_p(x)]/f(x,\infty) = [1 - f_p(x)/f(x,\infty)][1-\exp(-\alpha t)] + [c/(x,\infty)][\exp(-\beta t) - \exp(-\alpha t)] \tag{21}$$

The actual values of $f(x,t)$ are generated by numerical data. The coefficients c, α, and β are found by using Eq. (21) in conjunction with the computer program CONMIN,[13] which minimizes the objective function

$$[f(x,t) - f_p(x,0)]/f(x,\infty) - [1 - f_p(x)/f(x,\infty)] \times [1-\exp(-\alpha t)] - c/f(x,\infty)[\exp(-\beta t) - \exp(-\alpha t)]$$

over the computed range of t. For the pressure coefficient

$$f_p(x) = \mp (2\alpha/M_\infty) \tag{22}$$

and for the shock motion

$$f_p = 0 \tag{23}$$

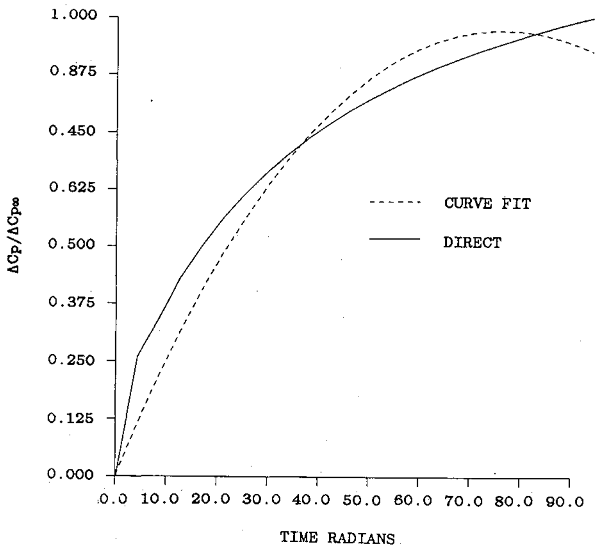

Fig. 1 Comparison of curve-fitted and direct calculations of $\Delta C_p/\Delta C_{p\infty}$ at $x=0.38$.

The program for the curve fit was based on an early version provided by Dr. Samuel C. McIntosh of McIntosh Structural Dynamics, Inc. The curve fit for the pressure perturbation $\Delta C_p(t)/\Delta C_p(\infty)$ at a value of x of 0.38 is shown in Fig. 1 where ΔC_p is the indicial response minus the value at the first time step. This effectively excludes the piston theory limit. The accuracy of the curve fit is quite good.

Model of Gust Alleviation

A model problem was developed to test the applicability of the indicial method to model gust alleviation. The problem consists of a "top-hat" gust that crosses the leading edge at time t_{10} and two control surfaces. For generality of computation, the sharp edge of the gust is replaced by a continuous variation similar to that of the control surfaces as given below. The characteristic times of the control surface are t_{10}, t_{11}, t_{12}, and t_{13}. The simple control law used in the following analysis was developed to illustrate an application of the idea; it should not necessarily be regarded as a practical device. The times t_{2i} $(i=0,3)$, etc., used in the calculation are similarily used only to illustrate the concept and should not be regarded as the characteristic times of an actual control.

At $t=t_{20}$, the control surface moves according to the law

$$\alpha = \alpha_0[1-\cos\omega_{21}(t-t_{20})]/2, \quad t_{20} \le t \le t_{21} \tag{24}$$

where ω_{21} is given by

$$\omega_{21} = \pi/(t_{21} - t_{20}) \tag{25}$$

At time t_{21}, the control stops. At time t_{22}, the control moves according to the law

$$\alpha = \alpha_0[\cos\omega_{22}(t-t_{22}) + 1]/2, \quad t_{22} \le t \le t_{23} \tag{26}$$

where

$$\omega_{22} = \pi/(t_{23} - t_{22}) \tag{27}$$

The second control surface moves in an analogous manner on the first control.

The control motion described by Eq. (24) has zero angular velocity at $t=t_{20}$ and t_{21}, which seems to be a realistic assumption. A similar behavior occurs at t_{22} and t_{23}.

In the indicial formulation, a typical quantity is given by

$$f(t) = f_\epsilon(t)\epsilon(0) + \int_0^t f_\epsilon(\tau)\frac{d\epsilon(t-\tau)}{d\tau}d\tau \tag{28}$$

where $f_\epsilon(t)$ is the indicial response. For the control laws described, $f(t)$ is given by

$$f(t)=\frac{\alpha_0}{2\omega}\int_{\hat{t_0}}^{\hat{t_1}} f_\epsilon(\tau)\,\sin\omega(t-\tau+t_0)\mathrm{d}\tau \tag{29}$$

where the generic control law

$$\alpha=\alpha_0[1-\cos\omega(t-t_0)]/2 \tag{30}$$

is used.

If the control surface movement starts at t_0 and finishes at t_1,

$$\frac{\mathrm{d}\epsilon(t-\tau)}{\mathrm{d}\tau}=0 \qquad \text{for} \quad t-\tau+t_0<t_0$$
$$t-\tau+t_0>t_1 \tag{31}$$

or, for a nonzero value of the integrand,

$$t+t_0-t_1\leq\tau\leq t \tag{32}$$

Also, for the general limits on the integral,

$$t_0\leq\hat{t_0} \qquad \hat{t_1}\leq t \tag{33}$$

Hence, the limits are

$$\hat{t_0}=\max[t_0,t+t_0-t_1]$$
$$\hat{t_1}=t \tag{34}$$

Control Optimization

Calculations using the indicial response require very little computer time. Thus, it is economical to couple the gust response and the control model to an optimizer to determine the best control deflections and to minimize the objective function F, where

$$F=\Delta C_L^2+\omega\Delta C_M^2 \tag{35}$$

There, ΔC_L and ΔC_M are the incremental lift and moment coefficients and ω a specified weighting function. The moments are about the quarter chord. The variables used in the optimization are the times t_{jk} ($j=2,3$; $k=0,1,2,3$) described in the previous section and the amplitudes of the control surface deflections. Constraints on t_{jk} are enforced; namely, that the control surface cannot operate until the gust has passed by 0.1 time unit and that the finish time of a control deflection cannot occur before the start time of the deflection.

The objective of this exercise is to devise simple control systems to counter the gust. The computer code CONMIN is used for the optimization.

Gust Response

In linear subsonic flow, the gust response is a function only of the freestream Mach number and the gust characteristics. In nonlinear transonic flow, the response is also a function of the steady flow over the airfoil and, hence, the airfoil geometry. This coupling manifests itself in two ways. First, the asymptotic lift of the wing due to a step in angle of attack α can be much greater than the classic value of $2\pi\alpha/\beta$, where

$$\beta=(1-M_\infty^2)^{1/2}$$

Second, the characteristic time of the response, that is, the transient behavior, is different from that in subsonic flow. For an airfoil to have a low gust response, the asymptotic lift increment of C_L due to the gust should be small. Also, the

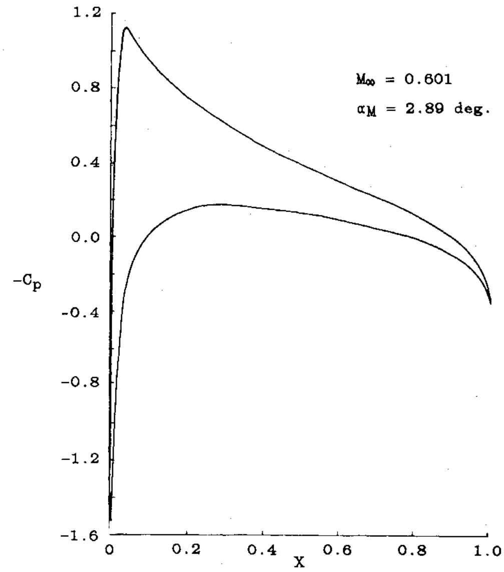

Fig. 2 Steady-state pressure distribution for a NACA 0012 airfoil.

characteristic time should be large to allow for a control to operate. For a gust alleviation device, the incremental lift ΔC_L should be large so that the control is effective; the characteristic time should be small so that the effect of the control occurs as early as possible.

In transonic flow, there is a nonlinear coupling between the steady and unsteady flows that possibly could be used to some advantage. Consider that the unsteady lift coefficient C_L is represented as follows. For a gust

$$C_L=\bar{f}_\infty[1-e^{-\bar{c}(t+t_0)}] \tag{36}$$

where $\bar{f}_\infty$ is the value of C_L as $t\rightarrow\infty$, $\bar{c}$ a constant, and the gust starts at $t=-t_0$. For a control surface

$$C_L=f_\infty+(f_p-f_\infty)e^{-ct} \tag{37}$$

where f_∞ is the value of C_L at $t\rightarrow\infty$ and f_p the value for piston theory at $t=0$.

Let the control law be

$$\alpha=(\alpha_0/2)(1-\cos\omega t) \qquad 0\leq t_1\leq t$$
$$\alpha=\alpha_0 \qquad t\geq t_1 \tag{38}$$

and

$$\omega t_1=\pi$$

Using Eqs. (36–38) in the indicial equations gives the total unsteady lift as

$$C_{L_T}=f_\infty\{1+g+e^{-ct}[(1+e^{\gamma\pi})(F-1)][2(1+\gamma^2)]-ge^{-\bar{c}t_0}\} \tag{39}$$

where

$$g=\bar{f}_\infty/f_\infty \qquad \gamma=c/\omega \qquad F=f_p/f_\infty \tag{40}$$

For C_{L_T} to be zero for all $t>t_0$,

$$g=-1$$
$$F=1+2\{[ge^{-\bar{c}t_0}(1+\gamma^2)]/(1+e^{\gamma\pi})\} \tag{41}$$

or, using Eq. (41),

$$F=1\{[2e^{-\bar{c}t_0}(1+\gamma^2)]/(1+e^{\gamma\pi})\} \tag{42}$$

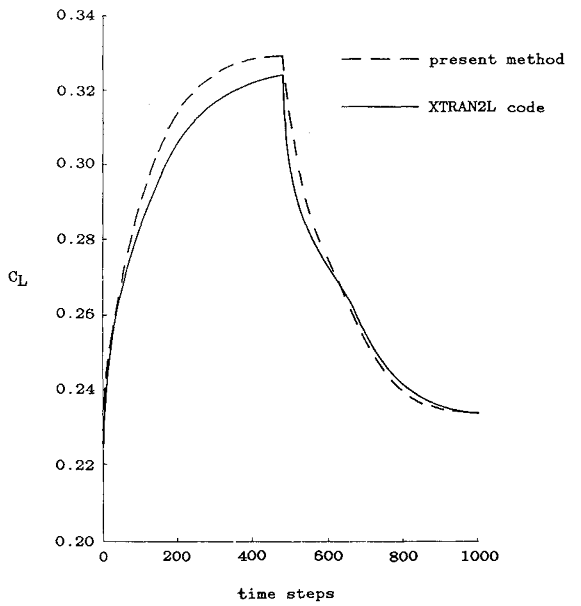

Fig. 3 Test case comparison between present method and XTRAN2L for a NACA 0012 airfoil at $M_\infty = 0.601$ and $\bar{\alpha}_m = 2.89$ deg.

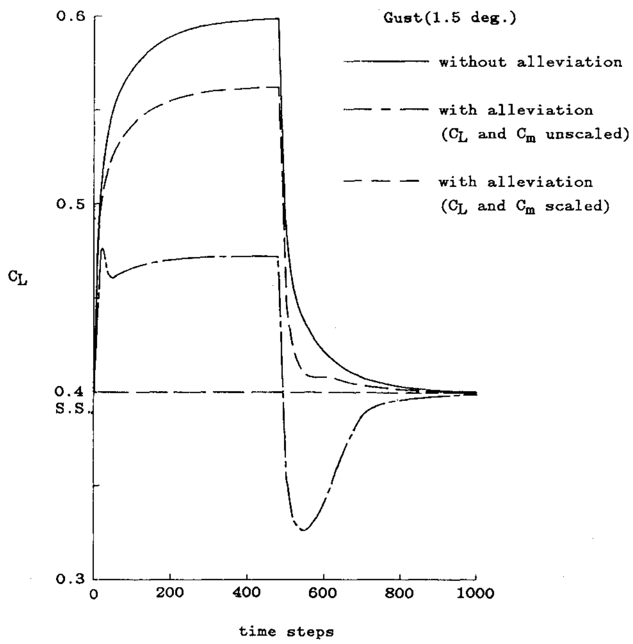

Fig. 4 C_L comparison between gust with and without control surface deflection alleviation for a NACA 0012 airfoil at $M_\infty = 0.601$ and $\alpha_m = 2.89$ deg.

For linear subsonic flow at a fixed Mach number, f_p, f_∞, and c are independent of airfoil section; ω is specified. Hence, there is no mechanism by which Eq. (42) can be satisfied in general. However, for transonic flow the quantities f_∞ and $\bar{c}$ depend on airfoil geometry, and it is possible that a certain design of airfoil and control surface could satisfy Eq. (42), leading to an effective gust alleviation device.

Table 1 Values of T_{rj}

T_{rj}	0	1	2	3
1	0.0	4.36×10^{-5}	50.0	$50 + 4.36 \times 10^{-5}$
2	0.0	22.0	60.0	82.0
3	0.0	22.0	60.0	82.0

Note: $r = 1$, gust; $r = 2$, trailing-edge control; and $r = 3$, leading-edge control.

Results

The computer code XTRAN2L is used to compute the necessary indicial responses and to compute fully nonlinear cases for comparison. In the following cases, the gust is of a top-hat geometry, starting at $t = 0$ and ending at $t = 50$. There are two control surfaces, each of 10% chord extent. One control is at the leading edge; the other is at the trailing edge. In the test cases, the controls operate at the time shown in Table 1. For the optimization cases, the gust amplitude is 1.5 deg.

The steady pressure distribution around a NACA 0012 airfoil at $\alpha_m = 2.89$ deg and $M_\infty = 0.601$ is shown in Fig. 2. This is a subcritical example. In Fig. 3, the prediction of the lift for this steady configuration is shown with a top-hat gust with amplitude equivalent to 0.25 deg and two control surfaces deflected at 0.25 deg. The agreement with the direct result is quite good.

In Figs. 4 and 5, the incremental lift and moment coefficients for the gust are shown. The function F in Eq. (35) is minimized through the overall time, and the optimized incremental lift and moment coefficients are also shown. In the optimization, the magnitude of the lift coefficient was much greater than that of the pitching moment, and the function F represents essentially only the lift increment. Accordingly, to make both lift and moment terms of comparable size, the incremental lift and moment were scaled with respect to the peak lift and moment increments for the gust alone. These results are also shown in Figs. 4 and 5. It can be seen that in the present simple analysis, it is difficult for the two controls to alleviate both the lift and moment, although it is possible to control the lift itself quite well.

A similar series of data is shown in Figs. 6-9 for a NACA 64A006 airfoil at $\alpha_m = 1.0$ deg and $M_\infty = 0.825$. Again, the agreement between the direct result and the present result is good. In the optimization cases, similar conditions to those noted for the previous example apply. This is an example that has a supersonic bubble in the flow and a shock wave. It is interesting that it seems easier to control the gust response for C_L of a supercritical flow than a subcritical flow as can be observed in Fig. 4. However, it appears that it is easier to reduce C_M for the subcritical example. That the gust loading of the first transonic example can be alleviated more than the subsonic example reinforces the suggestion made earlier.

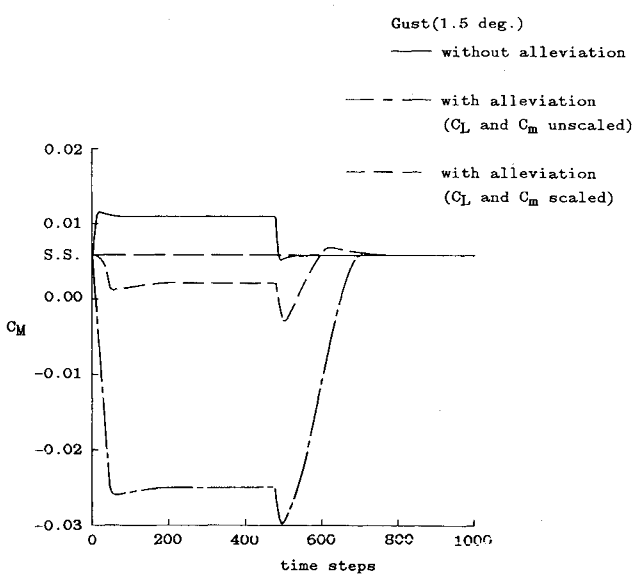

Fig. 5 C_M comparison between gust with and without control surface deflection alleviation for a NACA 0012 airfoil at $M_\infty = 0.601$ and $\alpha_m = 2.89$ deg.

In these examples, the lift and moment coefficients are linear functions of the various angles of deflection (Ref. 9).

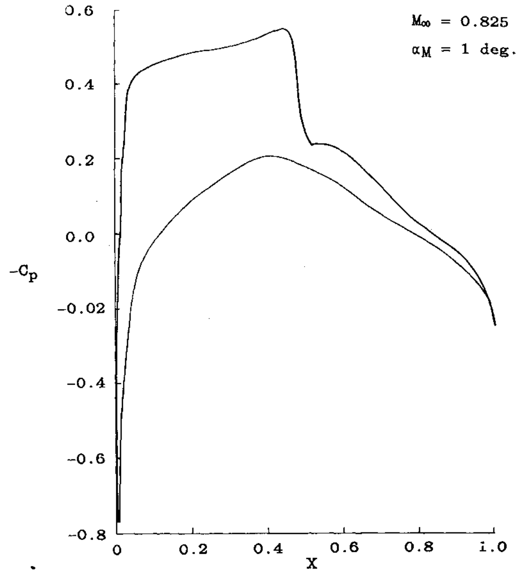

Fig. 6 Steady-state pressure distribution for a NACA 64A006 airfoil.

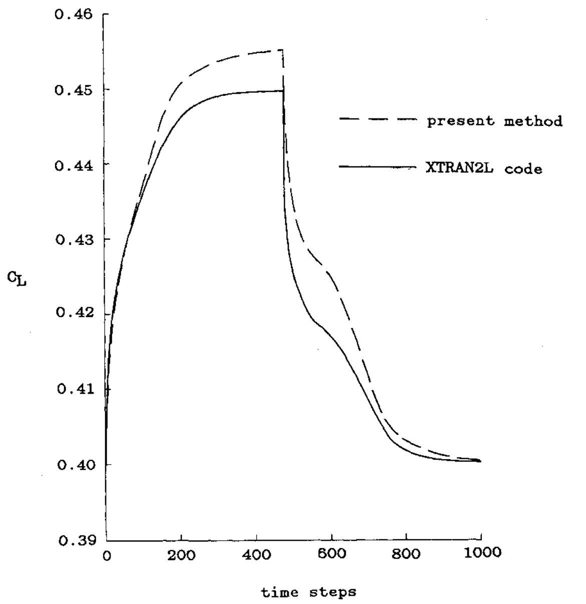

Fig. 7 Test case comparison between present method and XTRAN2L code for a NACA 64A006 airfoil at $M_\infty = 0.825$ and $\alpha_m = 1$ deg.

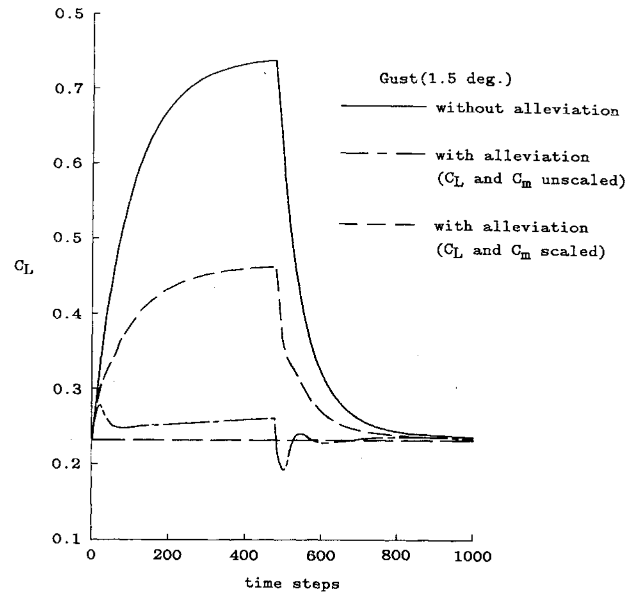

Fig. 8 C_L comparison between gust with and without control surface deflection alleviation for a NACA 64A006 airfoil at $M_\infty = 0.825$ and $\alpha_m = 1$ deg.

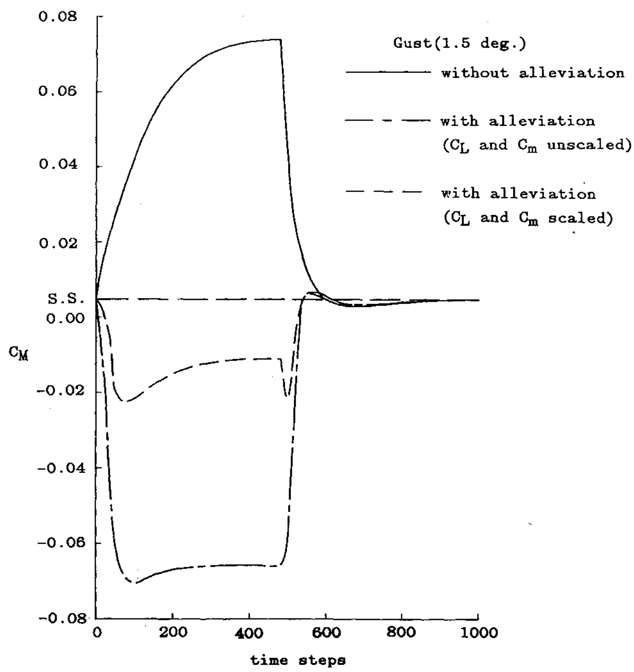

Fig. 9 C_M comparison between gust with and without control surface deflection alleviation for a NACA 64A006 airfoil at $M_\infty = 0.825$ and $\alpha_m = 1$ deg.

This is because the shock does not vanish on the airfoil. If the shock were to vanish, then the lift and moment coefficients would be nonlinear functions of the deflection angle. An example of nonlinearity is shown in Figs. 10 and 11, where the flow over a NACA 64A006 airfoil at $M_\infty = 0.875$, $\alpha = 0.0$ deg is also calculated for two different angles of deflection for the flap. For a 1/4-deg deflection, there is substantial shock movement on the flap causing the shock to appear on the flap only for a portion of the cycle. This gives rise to a nonlinear behavior of the hinge moment with respect to the flap deflection. The difference in hinge moment is considerable and can be seen in Fig. 10. The corresponding lift coefficients are shown for comparison in Fig. 11. The behavior is linear. In both Figs. 10 and 11, C_H and C_L for the 1/2-deg case are divided by 2 for comparison.

Possible Extensions of Work

The main advantage of the present technique is that the computational effort is minimal, thus allowing the use of an optimization routine to determine the best control movements. Provided the number of shock waves in the flow does not change, the method is very useful. If the number of shock waves does change during the unsteady motion, the present theory is invalid; however, it may be possible to extend the theory to treat this aspect. Such an extension for steady flow is given in Ref. 14. There is no difference in principle in the indicial theory between two and three dimensions, and a logical extension of the present work would be to three-dimensional flows.

Several aspects of the existing two-dimensional procedure need improving. One aspect concerns the problems that can arise when the supersonic bubble is small and near the leading edge. The present method uses only one point straining (at the

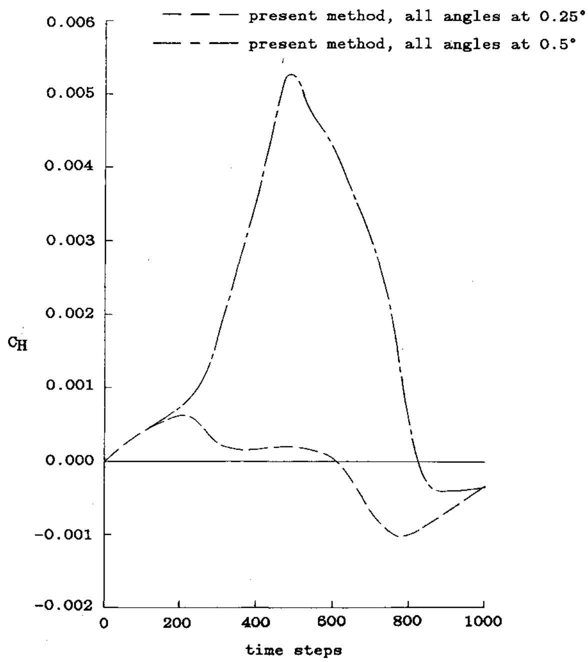

Fig. 10 Test case comparison of C_H for a NACA 64A006 airfoil at $M_\infty = 0.875$ and $\alpha_m = 0$ deg.

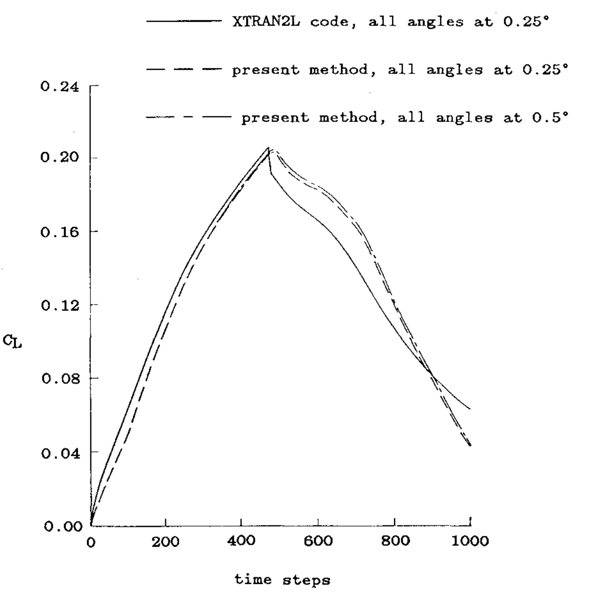

Fig. 11 Test case comparison of C_L for a NACA 64A006 airfoil at $M_\infty = 0.875$ and $\alpha_m = 0$ deg.

shock wave). It has been found in earlier work on the steady flow problem that the points characterizing large gradients also need to be strained. Such a modification is a straightforward task. A second aspect is that the optimization code CONMIN may not be the best code to use; further study of the possibilities of using a different optimization code is desirable.

Because of the computational speed of the present method, it can be used to investigate different methods of gust alleviation, including radical concepts. The main restriction on the present work is that the aerodynamics of such devices must be capable of being modeled by potential theory because, at present, unsteady transonic flows are not routinely calculated by the Euler or Navier-Stokes equations. If such simulations were available, the indicial method would be applicable. It should be noted that a Navier-Stokes simulation is similar in principle to experimental data.

Conclusions

A method for predicting the gust loading on an airfoil moving at transonic speeds based on the indicial theory has been developed that is computationally efficient. The method allows easy incorporation of gust alleviation devices and, when linked to an optimization method, gives the best mode of operation of these control devices. The method should be applicable to three-dimensional flows where a considerable savings in computer resources over a direct simulation of the unsteady flow could be obtained. The method is semianalytic in nature, which allows greater insight into methods of gust alleviation than can be obtained from a purely numerical study. Further study of the equations should lead to new ideas for alleviating the effect of gusts.

Acknowledgments

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